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NASA PROJECT APOLLO WORKING PAPER NO. 1078

AN APPROXIMATE METHOD OF CALCULATING THE ANGLE OF ATTACK RESPONSE OF THE C-1 VEHICLE DUE TO WINDS MEASURED PRIOR TO LAUNCH [U]

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
MANNED SPACECRAFT CENTER
Houston, Texas

October 18, 1963

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AN APPROXIMATE METHOD OF CALCULATING THE ANGLE OF ATTACK RESPONSE OF THE C-1 VEHICLE DUE TO WINDS MEASURED PRIOR TO LAUNCH [U]

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AN APPROXIMATE METHOD OF CALCULATING THE ANGLE OF ATTACK RESPONSE OF THE C-1 VEHICLE FROM WINDS MEASURED PRIOR TO LAUNCH

SUMMARY

The object of this paper is to derive a quick approximate method of calculating the angle of attack response of the C-1 to winds measured prior to launch. This is accomplished by reducing the equations of motion of the vehicle in the presence of a wind to a forced single degree of freedom system. The solutions are presented in a systematic tabular form and a sample wind profile is analyzed.

INTRODUCTION

The present procedure for obtaining the "α q" of a vehicle from the launch winds at Cape Canaveral is to teletype the wind data to a remote computer. There they are fed into a five-degree of freedom computer procedure and the resulting "α q" is determined. The maximum value thus obtained is compared with an allowable value and a "go" or "no-go" signal is sent back to the Cape.

It is the purpose of this paper to derive a quick, independent method of calculating the angle of attack response (and thus "α q") of the C-1 to launch winds. This approximate method will serve as a check of the computer analysis. Such a method was derived for the Mercury-Atlas flights by Mr. George A. Watts and Miss May T. Meadows of M.S.C. and this method is herein adapted to the C-1 vehicle.

This method of analysis is a useful tool for preliminary studies involving vehicle response to launch winds. Reference 1 is one example of the varied usage of this technique. In that reference, the method is used together with a statistical wind study to predict the monthly launch probabilities of the C-1 from Cape Canaveral.

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SYMBOLS

A	time varying coefficient
A_t	thrust minus axial drag, lb
a	coefficient in wind response matrix
B	time varying coefficient
C	time varying coefficient
C_{n_α}	aerodynamic normal force coefficient per unit α, $\frac{1}{\text{rad}}$
D	time varying coefficient
e	base of natural logarithm
F	external forces, lb
f	vector component, defined as used, ft/sec
$f_u(t)$	forcing function in u direction, ft/sec
$f_v(t)$	forcing function in v direction, ft/sec
g	acceleration due to gravity, ft/sec^2
h	vector component, defined as used, ft/sec
I	moment of inertia, ft-lb-sec
K	constant = 10 ft/sec
k	vector component defined as used, ft/sec
N_α	normal force per unit α, lb/rad.
$\ddot{n}$	acceleration normal to vehicle, ft/sec^2
q	dynamic pressure, psf
S	reference area, ft^2

[redacted]

T	thrust of gimbaled engines, lb
t	time, sec
u	$\Delta\dot{X}$ = change in $\dot{X}$ due to wind, ft/sec
u_g,	wind speed, ft/sec
V	vehicle forward velocity, ft/sec
v	$\Delta\dot{Y}$ = change in $\dot{Y}$ due to wind, ft/sec
W	vehicle weight, lb
X	distances along launch azimuth parallel to ground, ft
X_{cg}	distance from station 0 to center of gravity, ft
X_{cp}	distance from station 0 to center of pressure, ft
X_T	distance from station 0 to engine gimbal point, ft
y	distance normal to ground, ft
α	angle of attack, deg
β	angle between lateral velocity and vertical direction, deg
δ	root of equation
Θ	vehicle pitch attitude, deg
ξ	$\frac{gN_\alpha}{VW}$, $\frac{1}{sec}$
φ	engine deflection angle, deg

SUBSCRIPTS

o	initial value
A	Mode A
B	Mode B

[redacted]

n, n + 1, n - 1	layer numbers
r	resultant value
W	with wind

ASSUMPTIONS AND THEORY

Listed below are the important assumptions and approximations used in this paper.

1. Angle of attack due to the wind is approximated by wind speed times the cosine of the flight path angle divided by vehicle speed along the flight path.

2. The change in the horizontal and vertical components of vehicle velocity due to the wind are small compared to the vehicle's forward velocity.

3. Deviations from the programed attitude of the vehicle are negligible.

4. The time varying coefficients in the equations of motion do not vary significantly within altitude increments of 5,000 feet.

5. The wind profile is approximated by step functions taken in 5,000 foot increments, starting at a certain altitude and continuing to an infinite altitude, (see fig. 9).

6. The wind profile to be analyzed may be approximated adequately by 5,000 foot altitude increments.

7. The velocity along the flight path, and hence the dynamic pressure, is not appreciably affected by the wind.

THEORY: The accelerations in the X and Y directions on the C-1 launch vehicle in the presence of a wind (subscript W) in the pitch plane are given on the following page.

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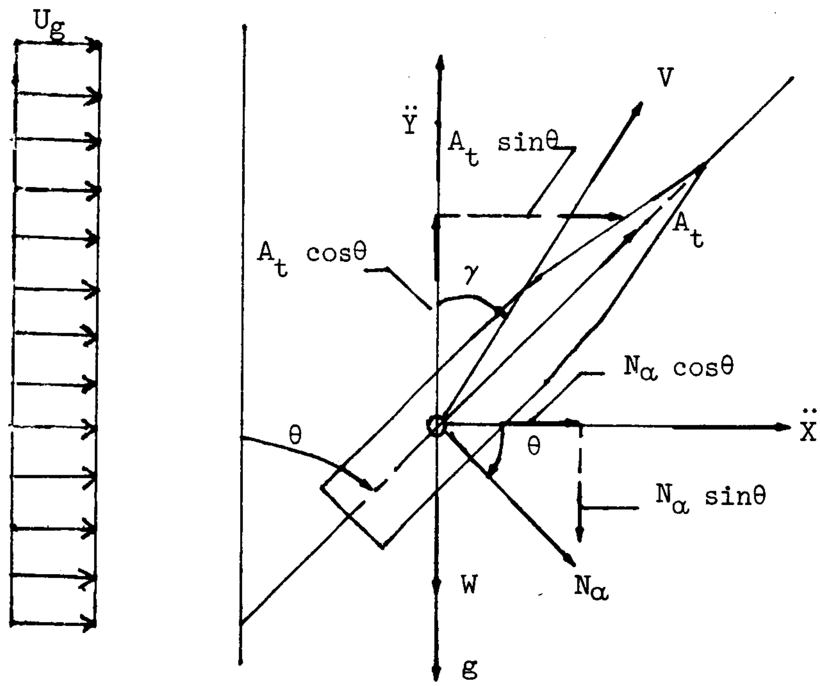

$$\ddot{X}_W = \frac{g}{W} A_t \sin \Theta + \frac{g}{W} N_\alpha \left(\Theta - \gamma_W + \frac{u_g}{V} \cos \gamma_W \right) \cos \Theta$$

$$\ddot{Y}_W = \frac{g}{W} A_t \cos \Theta - \frac{g}{W} N_\alpha \left(\Theta - \gamma_W + \frac{u_g}{V} \cos \gamma_W \right) \sin \Theta - g$$

Similar expressions can be written for the no wind case (no subscript):

$$\ddot{X} = \frac{g}{W} A_t \sin \Theta + \frac{g}{W} N_\alpha \left(\Theta - \gamma \right) \cos \Theta$$

$$\ddot{Y} = \frac{g}{W} A_t \cos \Theta - \frac{g}{W} N_\alpha \left(\Theta - \gamma \right) \sin \Theta - g$$

[redacted]

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Then to obtain the acceleration in the X direction due to the wind alone ($\Delta\ddot{X}$) it is only necessary to subtract the no wind accelerations from the accelerations with wind:

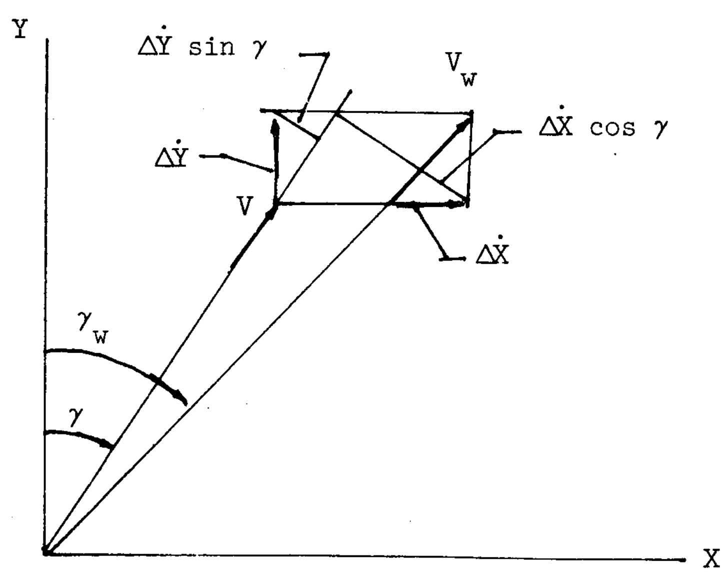
$$\Delta\ddot{X} = \ddot{X}_W - \ddot{X} = \frac{g}{W} N_\alpha \left[\frac{u_g}{V} \cos \gamma_W - \left(\gamma_W - W\right) \right] \cos \Theta \qquad (1)$$

Similarly for ΔY:

$$\Delta\ddot{Y} = \ddot{Y}_W - \ddot{Y} = \frac{g}{W} N_\alpha \left[\frac{u_g}{V} \cos \gamma_W - \left(\gamma_W - \gamma\right) \right] \sin \Theta \qquad (2)$$

These equations show that the horizontal and vertical accelerations due to the wind are a function of the wind speed and the deviation in flight path angle. In order to solve these differential equations it will be necessary to relate the deviation in flight path angle to the X and Y directions. An approximate relatiohsip is:

$$\gamma_W - \gamma = \frac{\Delta\dot{X} \cos \gamma - \Delta\dot{Y} \sin \gamma}{V}$$



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This expression is valid if $\Delta\dot{X}$ and $\Delta\dot{Y}$ are small compared to V. For this reason either V_W or V and γ_W or γ could be used in the equation. Since V and γ are known they are used and no subscripts will appear from here on. Making this substitution for $\gamma_W - \gamma$ in equations (1) and (2) yields:

$$\Delta\ddot{X} = \frac{g}{W} N_\alpha \cos \Theta \left[\frac{u_g}{V} \cos \gamma - \Delta\dot{X} \frac{\cos \gamma}{V} + \Delta\dot{Y} \frac{\sin \gamma}{V} \right]$$

$$\Delta\ddot{Y} = \frac{g}{W} N_\alpha \sin \Theta \left[- \frac{u_g}{V} \cos \gamma + \Delta\dot{X} \frac{\cos \gamma}{V} - \Delta\dot{Y} \frac{\sin \gamma}{V} \right]$$

This shows, unfortunately, that X and Y are not independent. However, since we are interested in velocities only, it is possible to reduce the equation to a first order differential equation with time varying coefficients forced by the wind velocities.

By letting $u = \Delta\dot{X}$, $v = \Delta\dot{Y}$ and A, B, C, D be the time varying coefficients, the equations reduce to:

$$\frac{du}{dt} = - Cu + Dv + f_u(t) \tag{3}$$

$$\frac{dv}{dt} = Au - Bv - f_v(t) \tag{4}$$

The $f_u(t)$ and $f_v(t)$ are the forcing functions. In order to solve this set of differential equations A, B, C, and D are assumed to be constant over a finite increment of time (altitude layer) and the solution for a typical increment follows. The complementary solution may be found by letting the forcing function equal zero:

$$\frac{du}{dt} + Cu - Dv = 0 \tag{5}$$

$$\frac{dv}{dt} - Au + Bv = 0 \tag{6}$$

[redacted]

Assuming the form of the solution to be:

$$u = u_o\, e^{\delta t}, \qquad v = v_o\, e^{\delta t}$$

Then:

$$\frac{du}{dt} = \delta u_o\, e^{\delta t}, \quad \frac{dv}{dt} = \delta v_o\, e^{\delta t}$$

So (5) and (6) become:

$$\delta u_o + C u_o - D v_o = 0 \tag{7}$$

$$\delta v_o - A u_o + B v_o = 0 \tag{8}$$

The above equations may be written as:

$$\begin{bmatrix} \delta + B & -A \\ -D & \delta + C \end{bmatrix} \begin{vmatrix} v_o \\ u_o \end{vmatrix} = 0$$

Solving for δ:

$$(\delta + B)\,(\delta + C) - AD = 0$$

$$\delta^2 + (B + C)\,\delta + BC - AD = 0$$

$$\delta = -\frac{(B + C)}{2} \pm \sqrt{\frac{(B + C)^2}{2} + AD - BC}$$

Where:

$$A \quad = \frac{gN_\alpha}{VW} \sin\Theta \cos\gamma$$

$$B \quad = \frac{gN_\alpha}{VW} \sin\Theta \cos \sin$$

$$C = \frac{gN_\alpha}{VW} \cos \Theta \cos \gamma$$

$$D = \frac{gN_\alpha}{VW} \cos \Theta \sin \gamma$$

$$f_u(t) = \frac{gN_\alpha}{VW} \cos \Theta \cos \gamma \; u_g(t)$$

$$f_v(t) = \frac{gN_\alpha}{VW} \sin \Theta \cos \gamma \; u_g(t)$$

Also let $\xi = \frac{gN_\alpha}{VW}$

Then:

$$\delta = -\frac{\xi}{2} (\sin \Theta \sin \gamma + \cos \Theta \cos \gamma)$$

$$\pm \sqrt{\frac{\xi^2}{4}(\sin \Theta \sin \gamma + \cos \Theta \cos \gamma)^2 + \xi^2(\sin \Theta \cos \gamma \cos \Theta \sin \gamma - \sin \Theta \sin \gamma \cos \Theta \cos \gamma)}$$

$$\delta = \frac{-\xi}{2} \left[(\sin \Theta \sin \gamma + \cos \Theta \cos \gamma) \pm (\sin \Theta \sin \gamma + \cos \Theta \cos \gamma) \right]$$

Therefore:

$$\delta = -\xi (\sin \Theta \sin \gamma + \cos \Theta \cos \gamma) = -\xi \cos (\Theta - \gamma)$$

And

$$\delta = 0$$

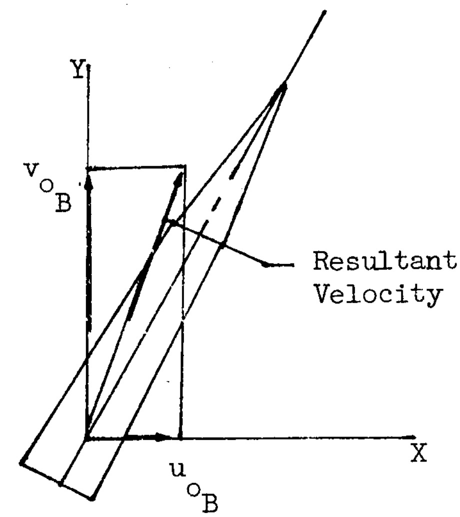
[redacted]

Then the complementary solutions are:

$$\left.\begin{aligned} u &= u_o\, e^{-\frac{gN_\alpha}{VW}\cos(\Theta - \gamma)t} \\ v &= v_o\, e^{-\frac{gN_\alpha}{VW}\cos(\Theta - \gamma)t} \end{aligned}\right\}\text{Mode A}$$

and for $\delta = 0$:

$$\left.\begin{aligned} u &= u_o \\ v &= v_o \end{aligned}\right\}\text{Mode B}$$



Since for the B mode $u = u_o$ and $v = v_o$, the resultant velocity will lie along the flight path. The A mode direction would normally be expected to be perpendicular to the B mode direction, however, it will be shown to be normal to the vehicle axis. This is indicated by the $\cos(\Theta - \gamma)$ term in the A mode solution.

Each mode is independent of the other and two independent differential equations may be found which can be forced independently. That is v_o is directly related to u_o so both u and v can be forced at the same time for each mode. This relationship can be found by substituting $\delta = -\xi(\sin\Theta\sin\gamma + \cos\Theta\cos\gamma)$ (Mode A) into equations (7) and (8):

[redacted]

$$\left[-\xi\,(\sin\Theta\sin\gamma + \cos\Theta\cos\gamma) + \xi\sin\Theta\sin\gamma\right] v_o - \xi\sin\Theta\cos\gamma\; u_o = 0$$

and

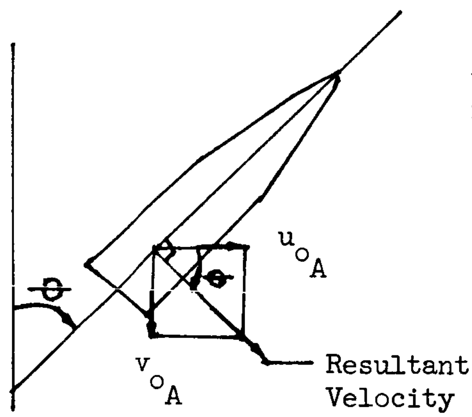
$$(-\xi\cos\Theta\sin\gamma)\, v_o - \xi\left[(\sin\Theta\sin\gamma + \cos\Theta\cos\gamma) - \xi\cos\Theta\cos\gamma\right] u_o = 0$$

$$-\cos\Theta\cos\gamma\; v_o - \sin\Theta\cos\gamma\; u_o = 0 \qquad (9)$$

$$-\cos\Theta\sin\gamma\; v_o - \sin\Theta\sin\gamma\; u_o = 0 \qquad (10)$$

thus from (9) and 10):

$$v_{o_A} = v_{o_A}\tan\Theta$$



where subscript A means A mode

In other words, the resultant velocity is normal to the vehicle axis.

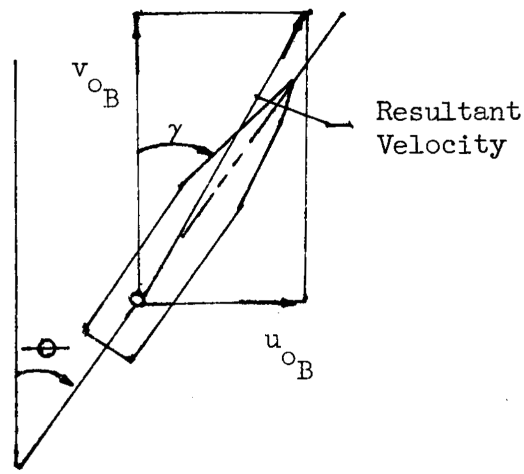
The relationship between u_o and v_o for mode B may be found by substituting $\delta = 0$ into equations (7) and (8):

$$Cu_o - Dv_o = 0$$

$$-Au_o + Bv_o = 0$$

or

$$\frac{v_{o_B}}{u_{o_B}} = \frac{A}{B} = \frac{C}{D} = \frac{\cos\gamma}{\sin\gamma} = \cot\gamma$$



Thus the resultant velocity vector for mode B is along the flight path.

Up to this point only the complementary solutions to the equations have been used. Now it is necessary to consider the forcing function. Examining the physical situation, it is seen that only accelerations normal to the vehicle due to the wind will affect vehicle response. Therefore, mode A is affected. Then u_A may be substituted for u and $-u_A \tan\Theta$ for v in equation (4):

$$-\tan\Theta \frac{du_A}{dt} - Au_A + B(-u_A \tan\Theta) = -f_v(t)$$

substituting for the coefficients:

$$\frac{du_A}{dt} + \frac{gN_\alpha}{VW}(\sin\Theta\sin\gamma + \cos\Theta\cos\gamma)\,u_A = \frac{gN_\alpha}{VW}\cos\Theta\cos\gamma\, u_{g(t)} \qquad (11)$$

This is the forced single degree equation for the vehicle in one layer. By letting $u_g(t) = K$ the particular solution of the equation is:

$$u_A = \frac{\cos\Theta\cos\gamma\,\frac{gN_\alpha}{VW}\,K}{\frac{gN_\alpha}{VW}(\sin\Theta\sin\gamma + \cos\Theta\cos\gamma)} = \frac{K}{\tan\Theta\tan\gamma + 1}$$

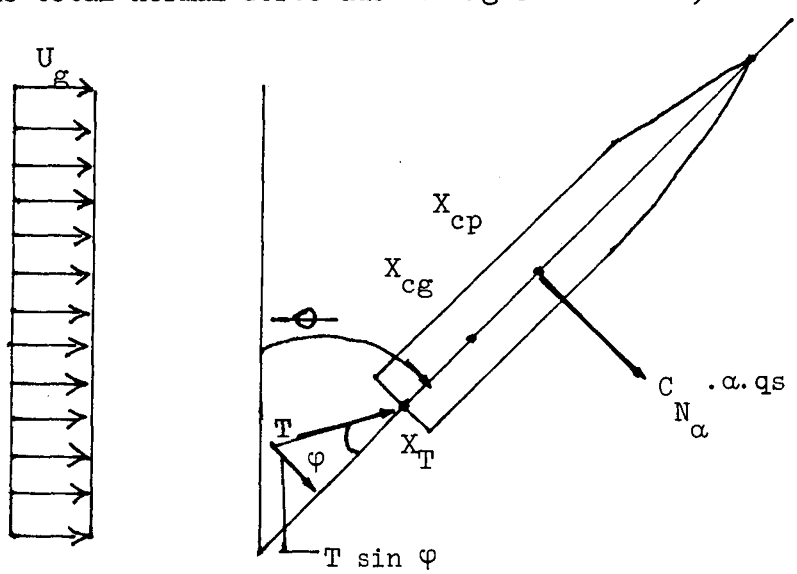
[redacted]

Thus the general solution for mode A:

$$u_A = u_{o_A} e^{-\frac{gN_\alpha}{VW} \cos(\Theta - \gamma)t} + \frac{K}{\tan \Theta \tan \gamma + 1} \qquad (12)$$

It will be assumed that $u_g(t)$ is a step function starting when the vehicle reaches a certain altitude and continuing to an infinite altitude. The vehicle response is desired. The coefficients are time varying, but they are assumed to be constant for 5,000 foot altitude layers. These constants change discontinuously at the ends of each layer. The values are for the middle of the altitude layer. The first layer is based on zero initial conditions. The speed at the end of the first layer determines the initial conditions for the second layer and so on up to 70,000 feet (the limit of this investigation). The initial conditions for the second layer are obtained by resolving u_A at the end of the first layer into the u_A direction of the second layer. The velocity in the B mode direction does not change throughout the layer but must be considered along with the A mode velocity at the end of the interval when finding the initial conditions for the third layer. This routine can be understood by following the example in the Assumptions and Procedure section of this paper.

N_α, the total normal force due to angle of attack, will now be evaluated:



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Sum of the forces normal to the vehicle:

$$\sum F = \frac{W}{g}\ddot{n} = C_{N_\alpha} \cdot \alpha \cdot qs + T \sin \varphi \tag{13}$$

Sum of moments about the C. G.:

$$\sum M = 0 = I\ddot{\Theta} = C_{N_\alpha} \cdot \alpha \cdot qs \left(X_{cp} - X_{cg}\right) - T \sin \varphi \left(X_{cg} - X_T\right) \tag{14}$$

Using $\varphi = \sin \varphi$ (small angle approximation) and solving for $T\varphi$ (14) becomes:

$$T\varphi = \frac{C_{N_\alpha} \cdot \alpha \cdot qs \left(X_{cp} - X_{cg}\right)}{\left(X_{cg} - X_T\right)} \tag{15}$$

Substituting (15) into (13):

$$\frac{W}{g}\ddot{n} = C_{N_\alpha} \alpha\, qs + \frac{C_{N_\alpha} \cdot \alpha \cdot qs \left(X_{cp} - X_{cg}\right)}{\left(X_{cg} - X_T\right)}$$

$$= C_{N_\alpha} \cdot qs \left[\frac{\left(X_{cg} - X_T\right) + \left(X_{cp} - X_{cg}\right)}{\left(X_{cg} - X_T\right)}\right] \alpha$$

or

$$N_\alpha = C_{N_\alpha} \cdot qs \left[\frac{\left(X_{cg} - X_T\right) + \left(X_{cp} - X_{cg}\right)}{\left(X_{cg} - X_T\right)}\right]$$

$$= C_{N_\alpha} \cdot qs \left[\frac{X_{cp} - X_T}{X_{cg} - X_T}\right] \tag{16}$$

[redacted]

[redacted]

PROCEDURE

Trajectory data for the C-1 as of 14 April 1962, are presented on figures 1 through 4. Figures 5 and 6 give data for the center of gravity (C.G.), center of pressure (C.P.) and the normal force coefficient due to angle of attack $\left(C_{N_\alpha}\right)$. These data along with the thrust gimbal point location (X_T = 100 in.) and a reference area ($S = 360$ ft^2) are used in the sample calculations that are to follow. N_α will be calculated by equation (16).

Then for the first layer, sea level to 5,000 feet:

$$N_{\alpha_{0-5}} = 6.4\ (70)\ (360)\ \left[\frac{618 - 100}{605 - 100}\right]$$

$$= 165{,}700\ \frac{\text{lb}}{\text{Rad}}$$

The N_α calculations are similar for all altitude layers so this sample will suffice. The N_α values are plotted together with the corresponding weights, both vs altitude, in figure 7.

Response calculations will now be performed for the first two altitude layers in a step by step procedure. The response to unit steps, originating at each 5,000 foot layer and continuing to 70,000 feet, is found. The analysis is then systematized into a tabular form and presented in the Results and Discussion section of this paper.

For the first layer:

$$t_1 = 0,\ u_A = 0$$

From equation (12):

$$0 = U_{o_A} + \frac{K}{\tan\Theta\tan\gamma + 1}$$

[redacted]

[redacted]

The final solution for the first layer is then (from (12)):

$$u_A = \frac{K}{\tan \Theta_1 \tan \gamma_1 + 1}\left(1 - e^{\frac{-gN_{\alpha_1}}{V_1 W_1} \cos (\Theta - \gamma)\, t_1}\right)$$

Using K = 10 ft/sec as a unit step, starting at sea level and extending to infinity:

For the first layer, 0 to 5,000 feet:

$$\left.\begin{aligned} K_1 &= 10 \text{ ft/sec} \\ \Theta_1 &= 3.2 \text{ deg} \\ \gamma_1 &= 2.3 \text{ deg} \\ N_{\alpha_1} &= 165{,}700 \text{ lb/rad} \\ V_1 &= 265 \text{ ft/sec} \\ W_1 &= 1{,}005{,}000 \text{ lbs} \end{aligned}\right\} \text{Mid-layer values from figure 1 through 4.}$$

$$u_{A_1} = \frac{10}{\tan 3.2^\circ \tan 2.3^\circ + 1}\left(1 - e^{\frac{-32.2 \times 165{,}700}{265 \times 1{,}005{,}000} (\cos .9^\circ)^{t_1}}\right)$$

$$= 9.97\left(1 - e^{-.02^{t_1}}\right)$$

at 5,000 feet t_1 = 27.6 seconds, then

$$u_{A_1} = 4.24 \text{ ft/sec}$$

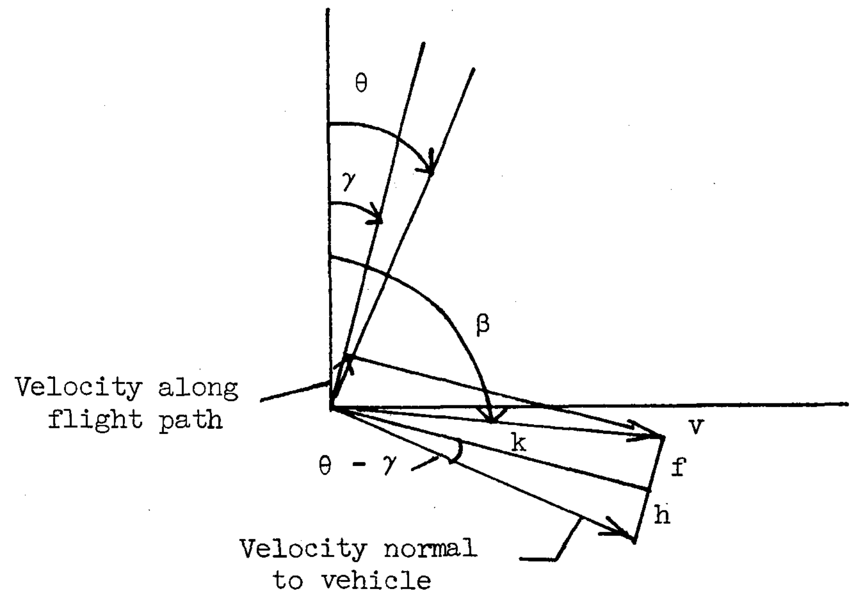
and $v_{A_1} = -u_{A_1} \tan \Theta$

$$= -4.24\ (.0559)$$

$$v_{A_1} = -.238 \text{ ft/sec}$$

[redacted]

For the second layer, 5,000 to 10,000 feet, the velocity vector due to the wind at the end of the first interval is normal to the vehicle body. In the second layer this velocity vector becomes the "initial" velocity and is resolved into two directions; one along the flight path (mode B which will be ignored until the third layer) and the other normal to the new vehicle attitude, Θ (mode A).



In general let v represent the velocity vector of the vehicle due to the wind at the beginning of a new altitude layer at angle β from the vertical.

Then:

$$f = v \sin (\gamma + 90^\circ - \beta)$$

$$e = v \cos (\gamma + 90^\circ - \beta)$$

Therefore velocity normal to the vehicle

$$= \frac{k}{\cos (\Theta - \gamma)}$$

$$h = k \tan (\Theta - \gamma)$$

and $\quad f + h = v \sin (\gamma + 90^\circ - \beta) + v \cos (\gamma + 90 - \beta) \tan (\Theta - \gamma) \qquad (18)$

$$= \text{velocity along flight path}$$

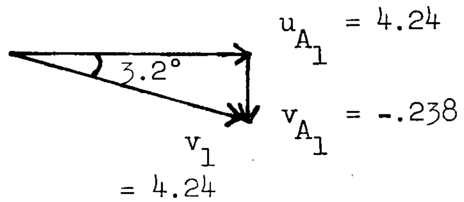
Since B mode velocity is assumed constant over the next interval (n + 1 interval), only the new value of A mode velocity must be found. The $u_{A_{t_n}}$ velocity component of the A mode velocity is the initial velocity of the n + 1 interval:

$$u_{oA_{t_{n+1}}} = \frac{k \cos \Theta}{\cos (\Theta - \gamma)} = \frac{v \cos (\gamma + 90 - \beta) \cos \Theta}{\cos (\Theta - \gamma)} \qquad (19)$$

For the second layer, 5,000 to 10,000 feet:

$$\left.\begin{aligned} K_2 &= K_1 = 10 \text{ ft/sec} \\ \Theta_2 &= 9.7° \\ \gamma_2 &= 8.8° \\ N_{\alpha_2} &= 588{,}000 \text{ lb/rad} \\ V_2 &= 508 \text{ ft/sec} \\ W_2 &= 924{,}000 \text{ lb} \end{aligned}\right\} \text{Mid-layer values from figure 1 through 4.}$$

And:



So $v_1 = 4.24$ ft sec and $\beta = 90 + 3.2 = 93.2°$

From (19):

$$u_{oA_{t_2}} = \frac{4.24 \cos (8.8 + 90 - 93.2) \cos 9.7°}{\cos (9.7° - 8.8°)} = 4.17 \text{ ft/sec}$$

Now considering B mode velocity for the first time: B mode velocity =

[redacted]

[redacted]

4.24 sin (8.8 + 90 - 93.2) + cos (8.8 + 90 - 93.2) tan (9.7 - 8.8) = .482 ft/sec. Then for this and subsequent layers the initial conditions at the beginning of each layer give:

$$u_a = u_{o_{A_{t_2}}} \text{ , } t = 0 \text{ (beginning of layer)}$$

then from (12):

$$u_{o_A} = u_{A_{t_2}} - \frac{K}{\tan \theta \tan \gamma + 1}$$

Therefore (12) becomes:

$$u_{A_2} = \left(u_{A_{t_2}} - \frac{K_2}{\tan \Theta_2 \tan \gamma_2 + 1} \right) e^{\frac{-gN_{\alpha_2}}{V_2 W_2} \cos (\vartheta_2 - \gamma_2)\, t_2} + \frac{K_2}{\tan \Theta_2 \tan \gamma_2 + 1}$$

$$u_{A_2} = \left(4.17 - \frac{10}{\tan 9.7^\circ \tan 8.8^\circ + 1} \right) e^{\frac{-32.2 \times 588{,}000}{508 \times 924{,}000} \cos (9.7 - 8.8)\, t_2} + \frac{10}{\tan 9.7^\circ \tan 8.8^\circ + 1}$$

Since $t_2 = 9.9$ sec:

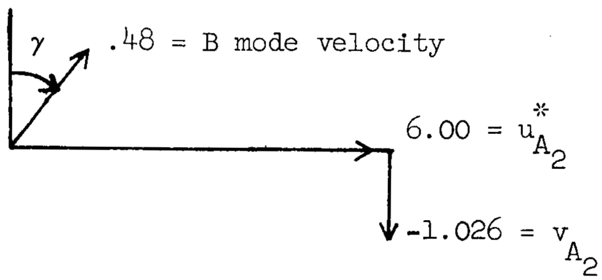
$$u^*_{A_2} = 6.00 \text{ ft/sec } (^*\text{means at end of layer})$$

$$v^*_{A_2} = u^*_{A_2} \tan \Theta_2 = -\ 6.00 \ (\tan 9.7^\circ) = -\ 1.026 \text{ ft/sec}$$

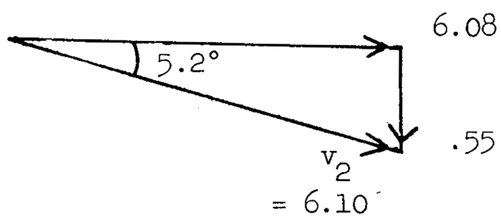
[redacted]

[redacted]

At this point v_2 and β should be evaluated.



Resolving into horizontal and vertical components:



$$\beta = 90 + 5.2 = 95.2°$$

For the third and subsequent layers a similar procedure is carried out. Thus a value of the net vehicle response is found from sea level to 70,000 feet for each 5,000 foot altitude layer. To convert this to angle of attack, v and β are resolved to the nominal trajectory at each altitude. The net angle of attack is then determined by:

$$\alpha = \frac{10}{V} \cos \gamma - \frac{v \sin (\beta - \gamma)}{V}$$

The γ and V values used here are for the ends of the layers.

The angle of attack for the two altitude layers previously considered are then:

α at 5,000 feet due to a 10 fps step starting at S.L.:

$$\alpha = \frac{10}{400} \cos 6.1° \frac{-4.24 \sin (93.2° - 6.1°)}{400} = .01423 \text{ rad} = .816 \text{ deg}$$

[redacted]

[redacted]

and α at 10,000 feet due to a 10 fps step starting at S.L.:

$$\alpha = \frac{10}{610} \cos 11.5° - 6.10 \sin \frac{95.2° - 115°}{610}$$

$$= \underline{.00614 \text{ rad.}} = \underline{.352 \text{ deg.}}$$

The angle of attack response to a 10 fps step starting at sea level has thus been calculated up to 70,000 feet and appears as the S.L. curve on figure 8. Similar curves are shown on this figure for 10 fps steps starting at each 5,000 foot level.

The triangular matrix presented in the Results and Discussion section of this paper is derived from this figure. The values tabulated are mid-layer values (read at 2,500 feet, 7,500 feet, et cetera).

As an example, the values tabulated in the first column of the matrix are for a 10 ft/sec wind step starting at sea level and are read from the sea level curve at the mid-layers; from figure 8:

Mid-layer altitude	Angle of attack response to 10 ft/sec step at S.L.
2,500	1.537
7,500	.545
12,500	.255

The following explanation illustrates how the triangular wind matrix is used to find the C-1 angle of attack response to a particular wind profile. If the wind profile can be broken into a series of vertical uniform profiles starting at each 5,000 foot level, then the responses to the unit profile factored by profile speed gives the response of the C-1 to the net wind when summed together (fig. 9).

Since the wind velocity given by the factored unit profiles is only correct at the center of each altitude layer, the responses will be assumed to be right there also.

[redacted]

 [redacted]

The general form of the matrix is:

$$\begin{bmatrix} a_{11} & & & \\ a_{21} & a_{22} & & \\ a_{31} & a_{32} & a_{33} & \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} \frac{u_1}{10} \\ \frac{u_2 - u_1}{10} \\ \frac{u_3 - u_2}{10} \\ \frac{u_4 - u_3}{10} \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{bmatrix}$$

Where a_{32} means angle of attack at the 3rd layer (12,500 feet) due to a unit 10 ft/sec wind starting at the 2nd layer (5,000 feet). The column of $\frac{u_n - u_{n-1}}{10}$ is the wind speed of the n layer minus the wind-speed of the n - 1 layer all divided by 10. The resultant angle of attack column is self explanatory. The angle of attack in this case is the response of the C-1 in the pitch plane and $\frac{u_n - u_{n-1}}{10}$ are of course wind components resolved into the pitch plane.

It should be noted that this procedure yields angle of attack due to the wind, the programed angle must be added to this to obtain the net angle of attack.

The sideslip angle due to the wind may be found in the same way. Since the slideslip component of the wind is usually small compared to the pitch plane component, errors induced by using the pitch plane matrix will not be very significant.

The resultant α is found by taking the square root of the sum of the squares of the net angle of attack and angle of sideslip. The "αq" is then found by multiplying the resultant α by the nominal "q". This is perhaps the biggest source of error in the method. Since "q" is generally smaller during a tailwind, this method usually gives a conservative value of "αq".

A step by step example will now be given for the wind profile shown on figure 9. It will be assumed that the vehicle will be launched directly downwind, otherwise an additional set of calculations would have

[redacted]

to be made for the angle of sideslip. Since the same procedure applies, this assumption will be made. First the wind speed will be read and the $\frac{u_n - u_{n-1}}{10}$ values calculated.

From figure 9:

Altitude layer, 1,000 ft	Mid-layer, ft	Mid-layer windspeed, u_n, fps	$\frac{u_n - u_{n-1}}{10}$ fps
0-5	2,500	56	5.6
5-10	7,500	69	1.3
10-15	12,500	81	1.2

Now the response of the C-1 to wind will be calculated. Refer to the table in the Results and Discussion section for "a" values.

Attitude layer,	Starting altitude of velocity step		
1,000 ft	0	5	10 . . .
0-5	$a_{11} \frac{(u_5 - u_o)}{10}$		
5-10	$a_{21} \frac{(u_5 - u_o)}{10}$	$a_{22} \frac{(u_{10} - u_5)}{10}$	
10-15	$a_{31} \frac{(u_5 - u_{10})}{10}$	$+a_{32} \frac{(u_{10} - u_5)}{10}$	$+a_{33} \frac{(u_{15} - u_{10})}{10}$
.	.	.	.
.	.	.	.
.	.	.	.

[redacted]

Then angle of attack due to wind $\left(\alpha_w\right)$ for:

Sea level to 5,000 feet

$$\alpha_{w_{o-5}} = a_{11} \frac{\left(u_5 - u_o\right)}{10} = 1.537\ (5.6) = \underline{8.607\ \text{deg}}$$

5,000 to 10,000 feet

$$\alpha_{w_{5-10}} = a_{21} \frac{\left(u_5 - u_o\right)}{10} + a_{22} \frac{\left(u_{10} - u_5\right)}{10}$$

$$= .543\ (5.6) + .885\ (1.3) = \underline{4.191\ \text{deg}}$$

10,000 to 15,000 feet

$$\alpha_{w_{10-15}} = a_{31} \frac{\left(u_5 - u_{10}\right)}{10} + a_{32} \frac{\left(u_{10} - u_5\right)}{10} + a_{33} \frac{\left(u_{15} - u_{10}\right)}{10}$$

$$= .255\ (5.6) + .43\ (1.3) + .65\ (1.2) = \underline{2.767\ \text{deg}}$$

and so on.

These responses due to the wind must be added to the programed angle of attack. Since sideslip has been assumed to be zero, this gives the resultant angle of attack which will be multiplied by the appropriate dynamic pressure to give "α q":

[redacted]

Altitude layer	α due to wind	Programed α	Resultant α	Nominal q	α q
0-5	8.607	.80	9.407	78	734
5-10	4.191	.50	4.691	244	1145
10-15	2.767	.20	2.967	392	1163
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.

Values of "α q" for this profile have been calculated up to 62,500 feet and are plotted on figure 10 vs altitude.

RESULTS AND DISCUSSION

Table I represents the results of this paper. Only this table and the wind profile to be studied are necessary to quickly calculate the angle of attack response of the C-1 vehicle (and hence "α q"). Since sample calculations for a specific wind profile have already been presented, no further explanation of usage will be given here.

This technique is useful for vehicle design studies, wind launch probability predictions and other associated problems. However, it should be emphasized that the assumptions made in deriving this method should be thoroughly understood before attempting to apply it to a particular vehicle.

REFERENCE

1. Wade, D. C.: "Apollo-Saturn C-1 Launch Probability Based on Winds Aloft". NASA Project Apollo Working Paper no. 1081, July 12, 1963.

[redacted]

TABLE I.- WIND RESPONSE MATRIX

Altitude layer ~ 1,000 ft	0	5	10	15	20	25	30	35	40	45	50	55	60	65
0-5	1.537													
5-10	.543	.885												
10-15	.255	.430	.650											
15-20	.138	.237	.363	.521										
20-25	.079	.146	.220	.321	.442									
25-30	.050	.092	.140	.201	.282	.381								
30-35	.030	.058	.093	.132	.192	.252	.342							
35-40	.020	.039	.060	.096	.140	.182	.242	.300						
40-45	.010	.025	.042	.070	.102	.133	.173	.222	.271					
45-50	.004	.018	.031	.049	.079	.100	.135	.170	.207	.250				
50-55	.002	.014	.025	.035	.060	.077	.105	.131	.161	.195	.221			
55-60	.000	.010	.020	.027	.048	.059	.084	.108	.132	.157	.180	.200		
60-65	.000	.008	.013	.022	.038	.050	.070	.087	.109	.130	.149	.165	.182	
65-70	.000	.004	.010	.030	.030	.041	.058	.071	.090	.108	.124	.139	.152	.163

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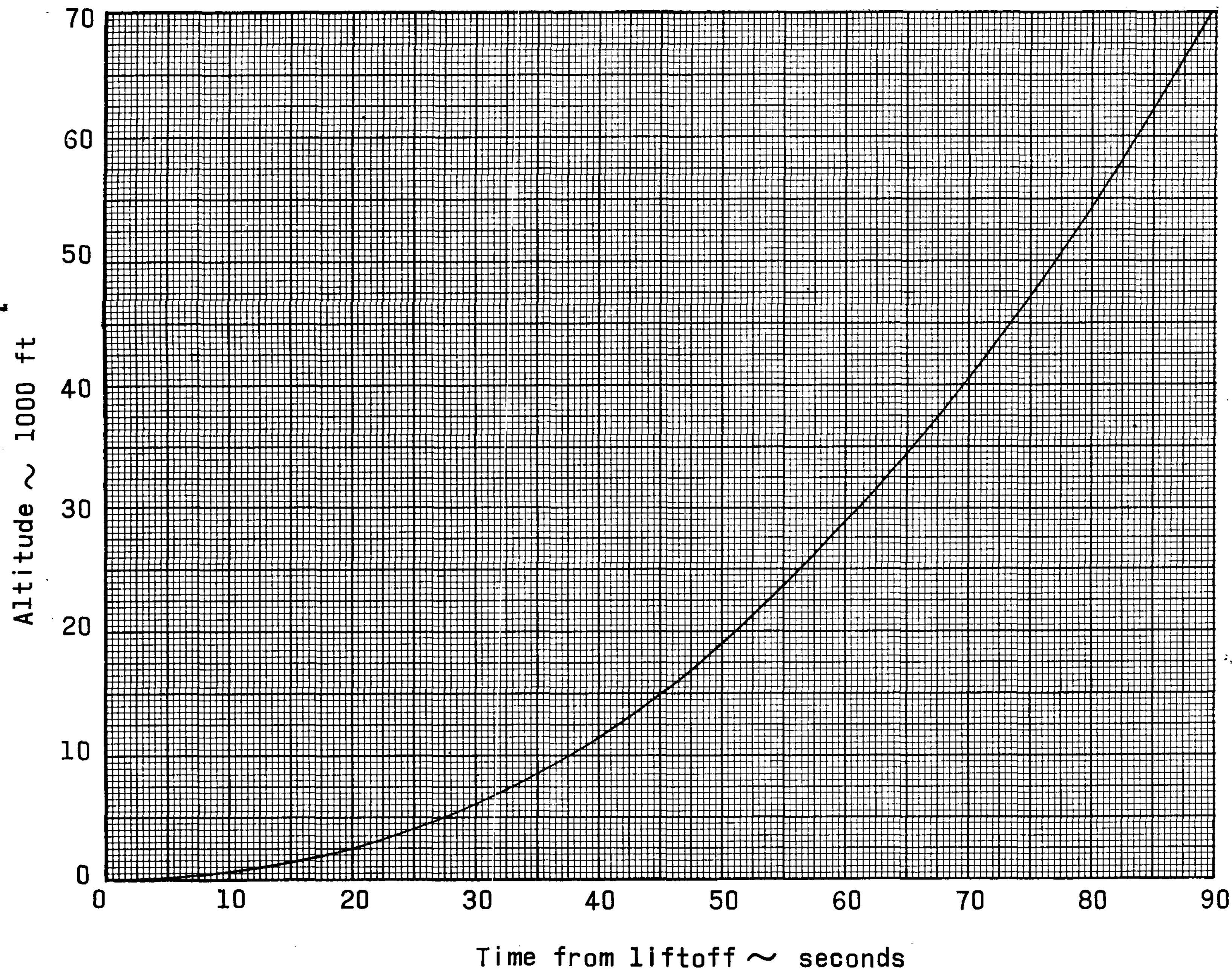

Figure 1.- Altitude versus time for a typical C-1 trajectory.

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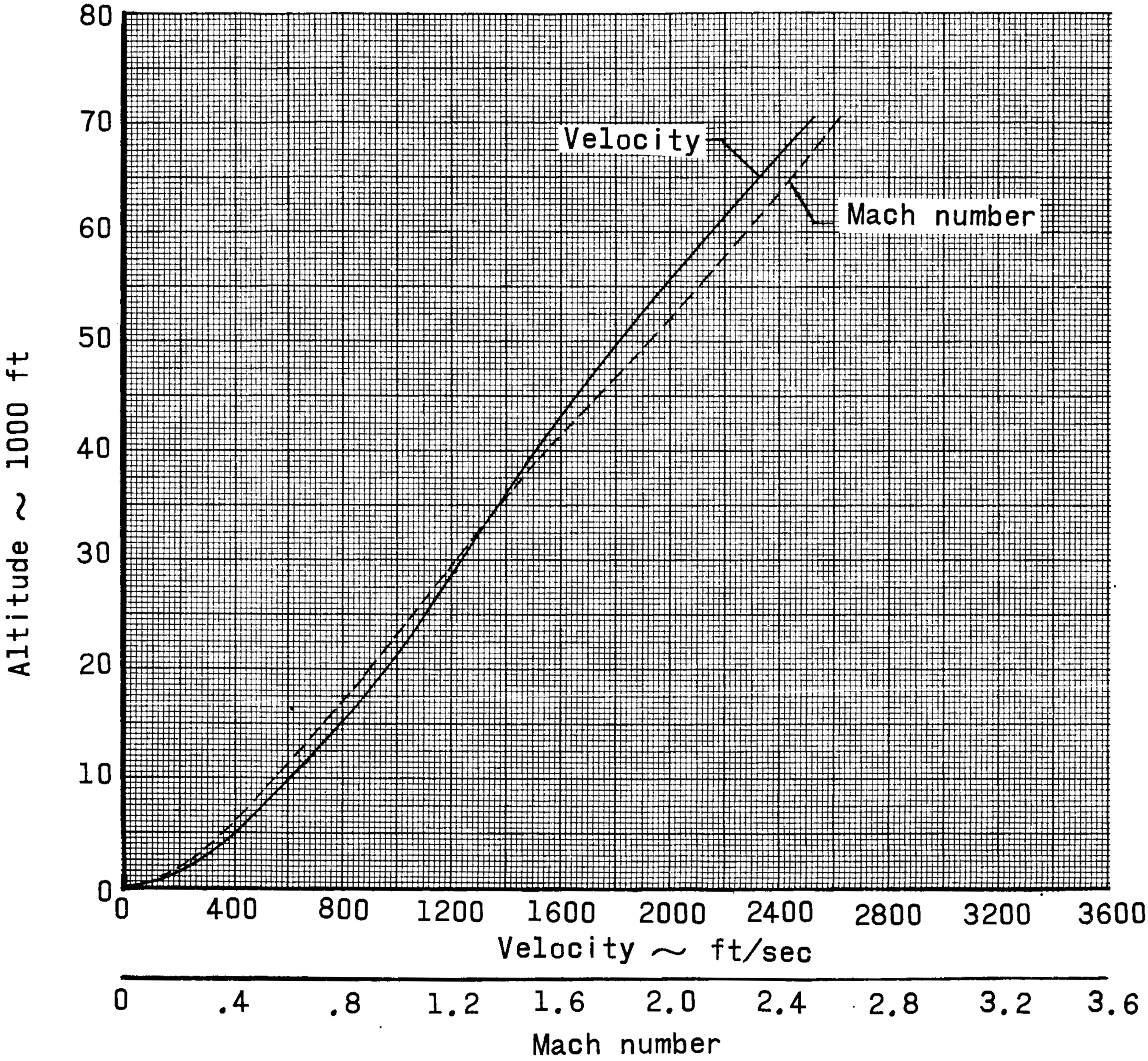

Figure 2.- Velocity and Mach number as a function of altitude for a typical C-1 trajectory.

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[redacted]

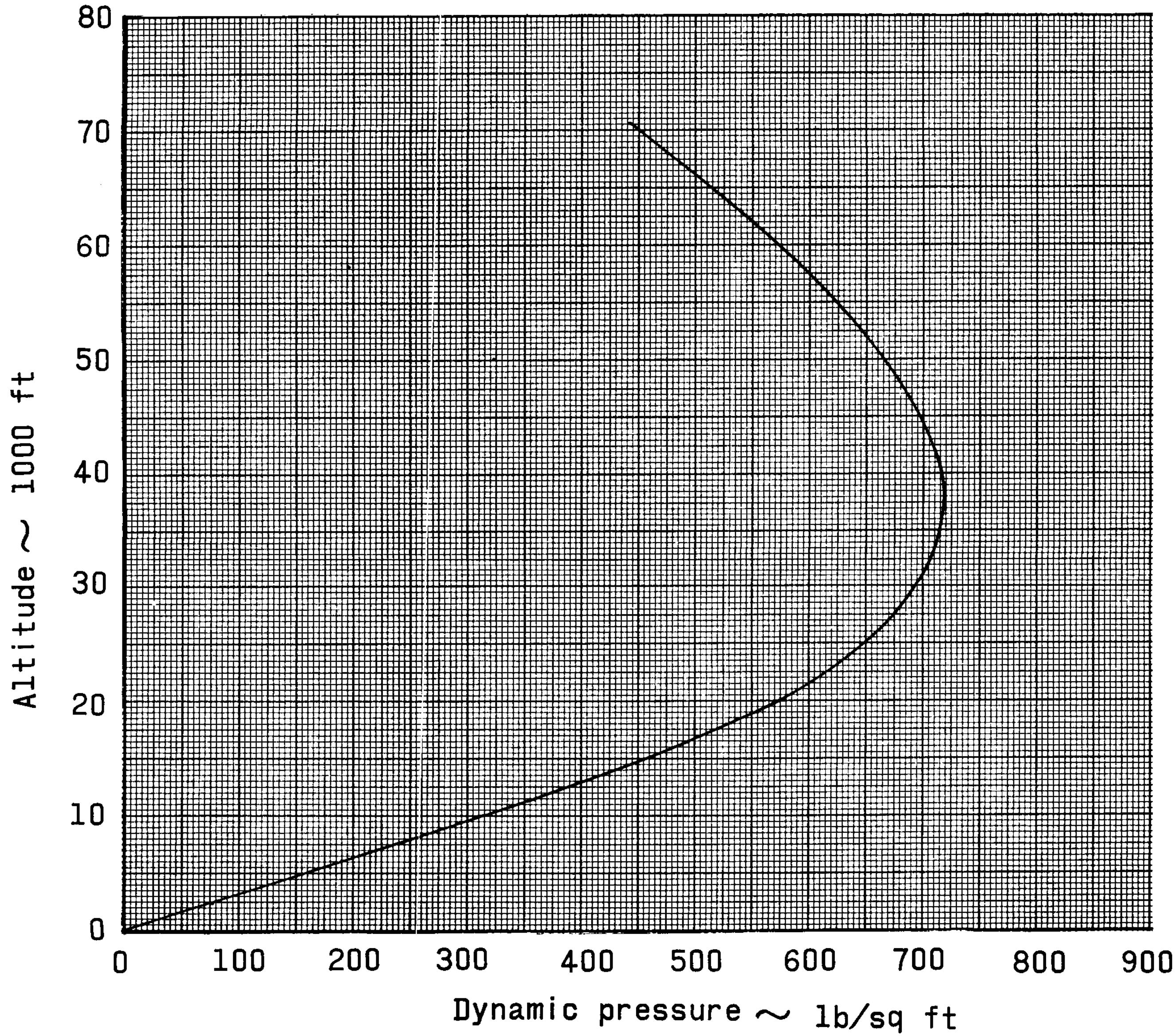

Figure 3.- Dynamic pressure versus altitude for a typical C-1 trajectory.

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[redacted]

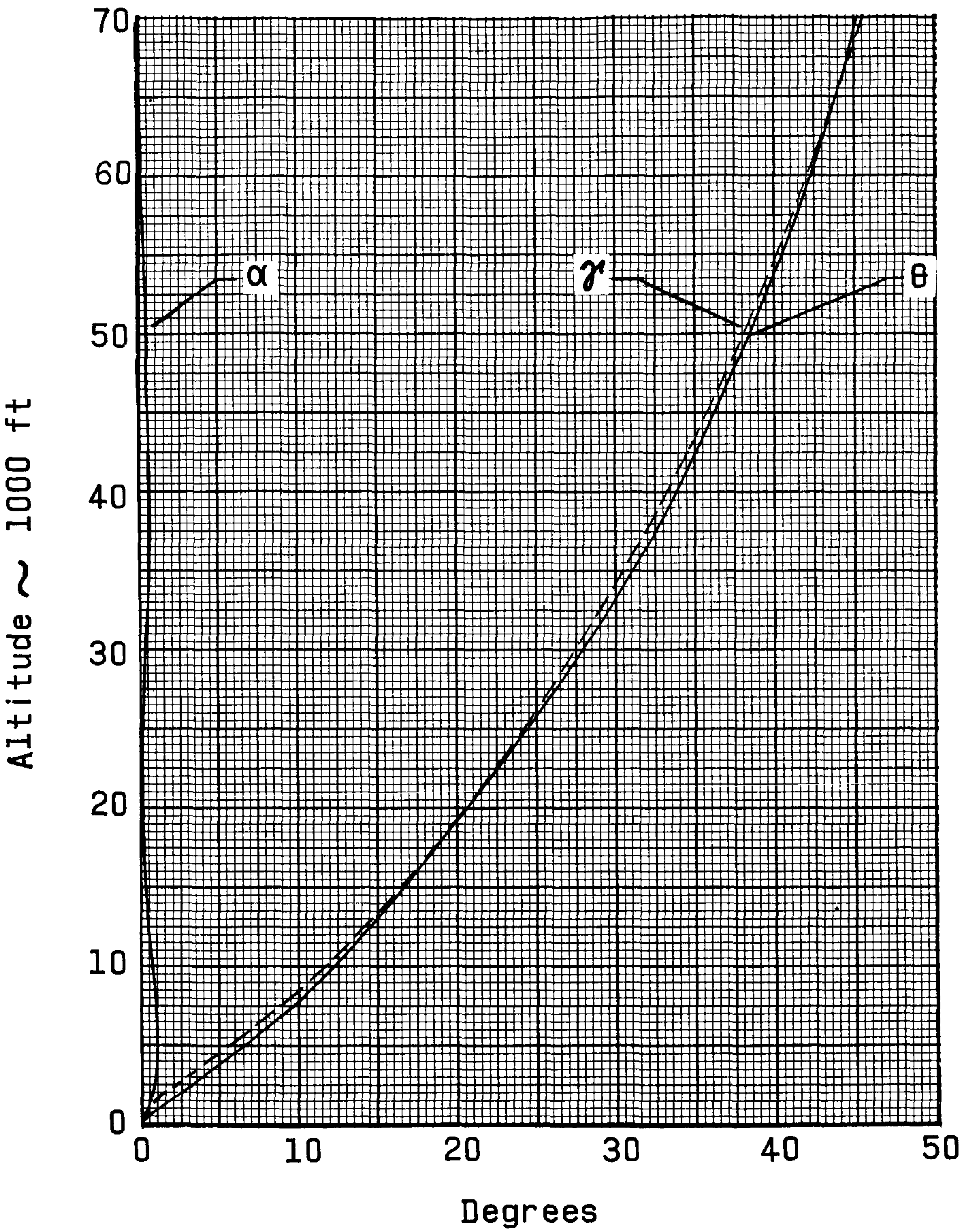

Figure 4.- Flight path angle, altitude and angle of attack as a function of altitude for a typical C-1 trajectory.

[redacted]

[redacted]

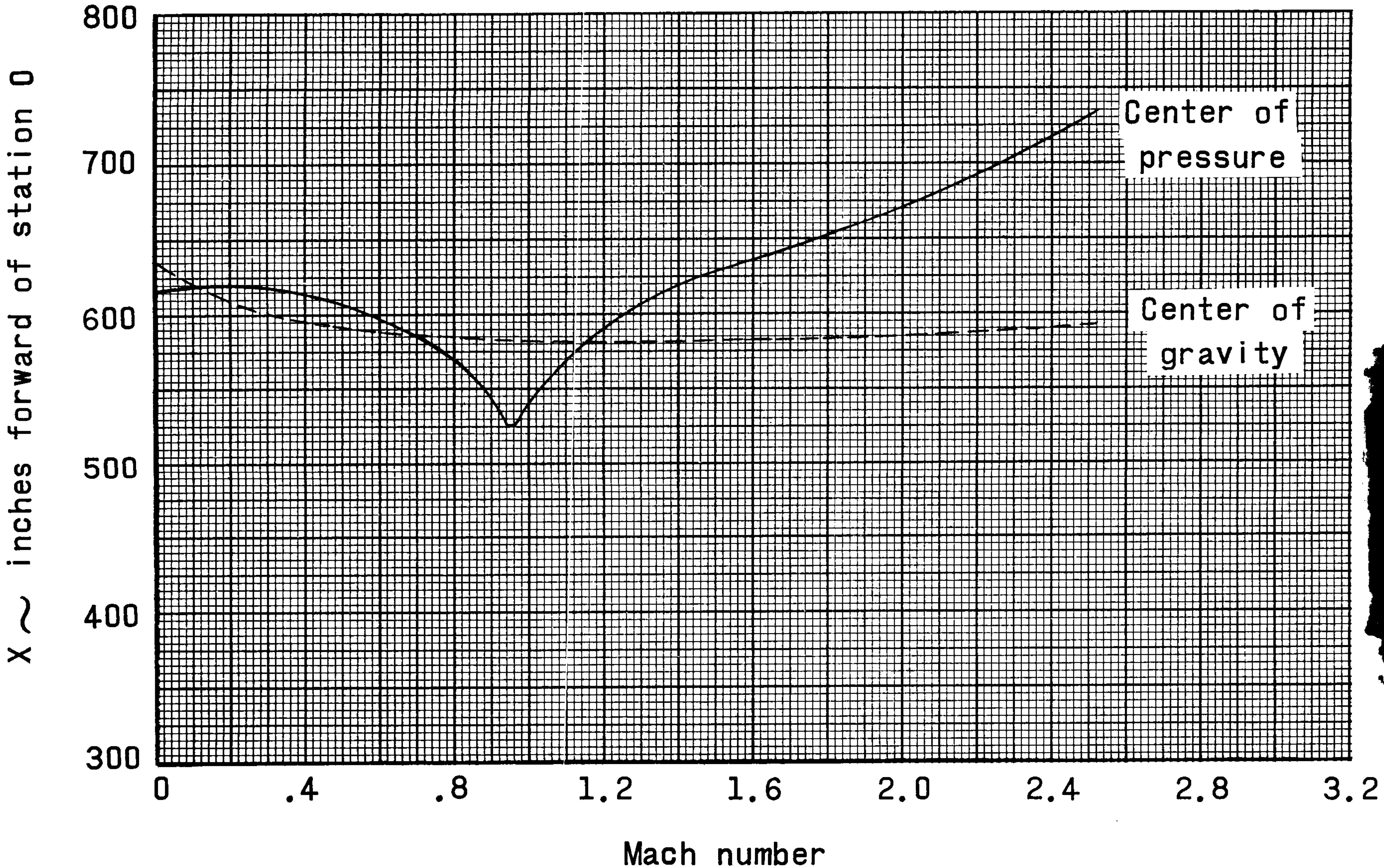

Figure 5.- Center of pressure and center of gravity versus Mach number for a typical C-1 trajectory.

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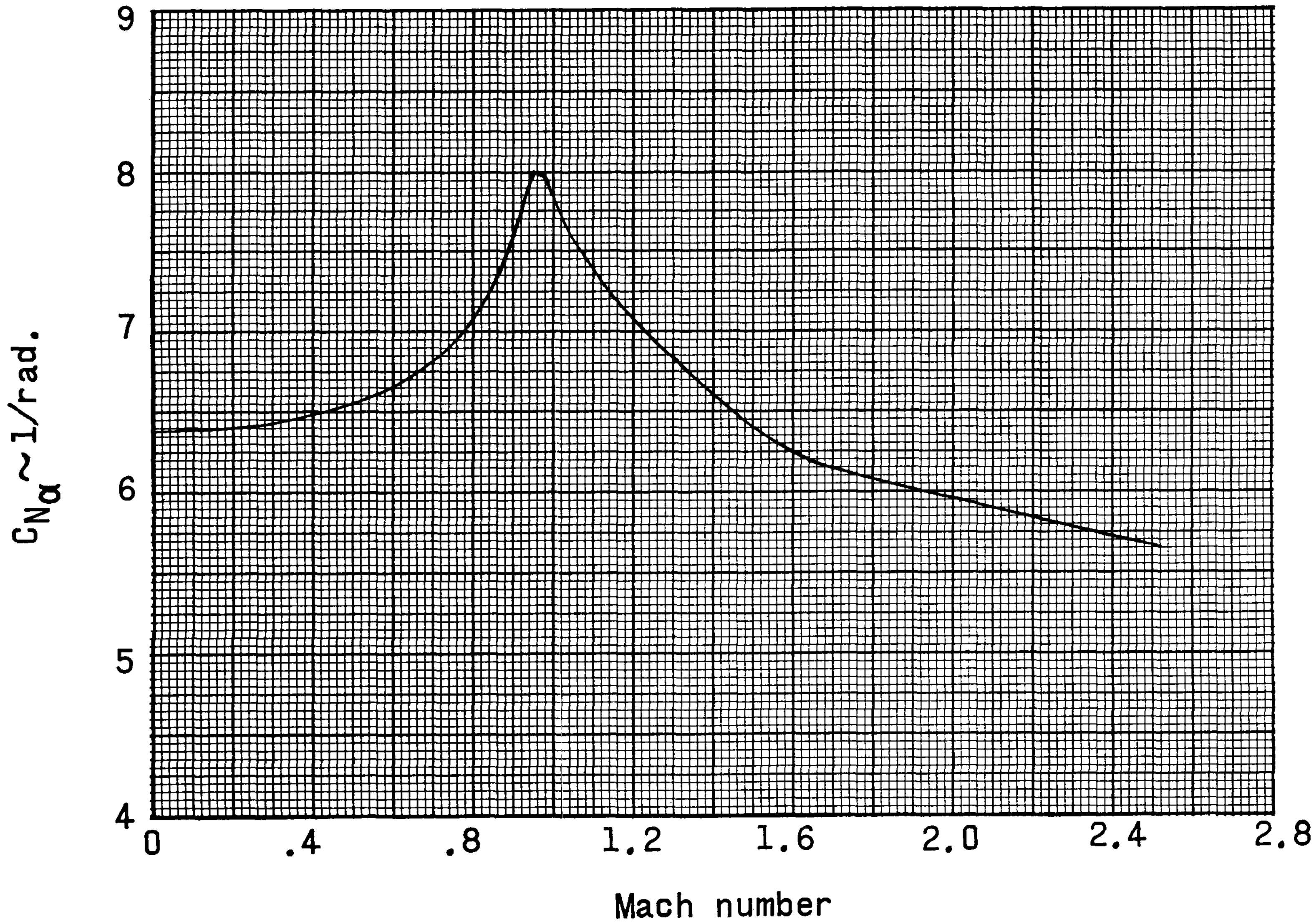

Figure 6.- Normal force coefficient per unit angle of attack versus Mach number.

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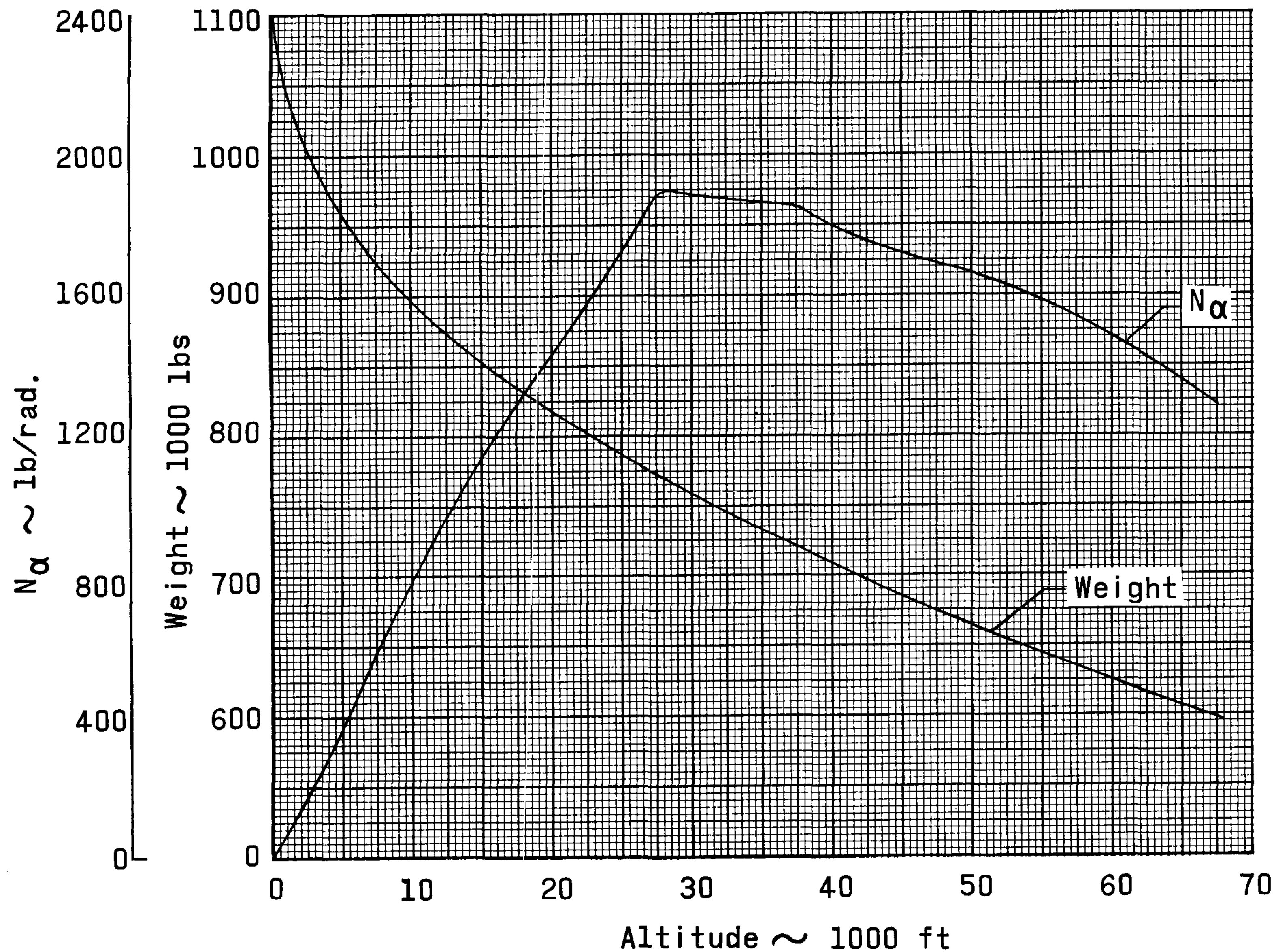

Figure 7.- Total normal force per unit angle of attack and vehicle weight as a function of altitude for a typical C-1 trajectory.

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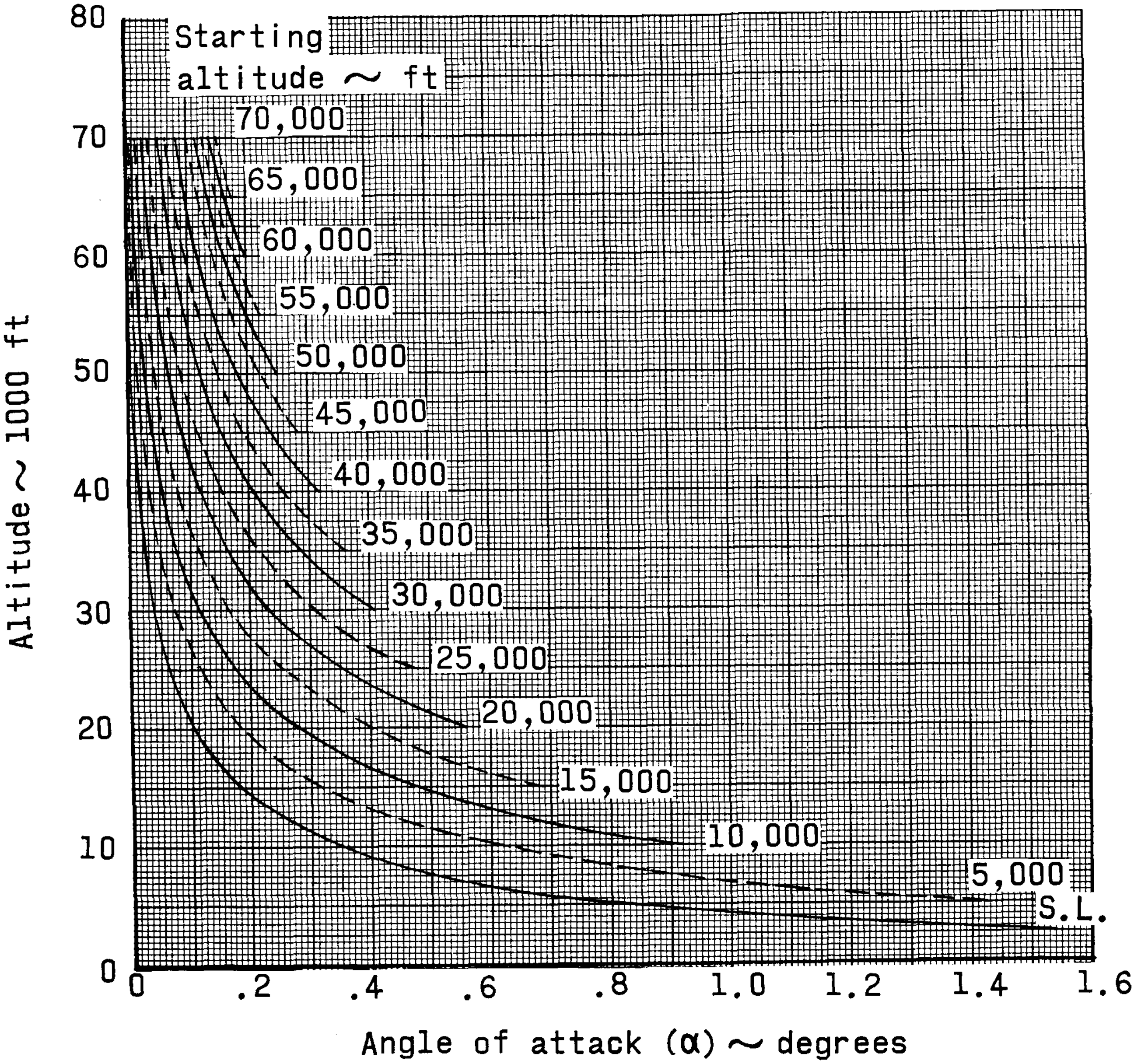

Figure 8.- Angle of attack response of the C-1 to a 10 feet per second step wind profile starting at various altitudes and continuing to infinity.

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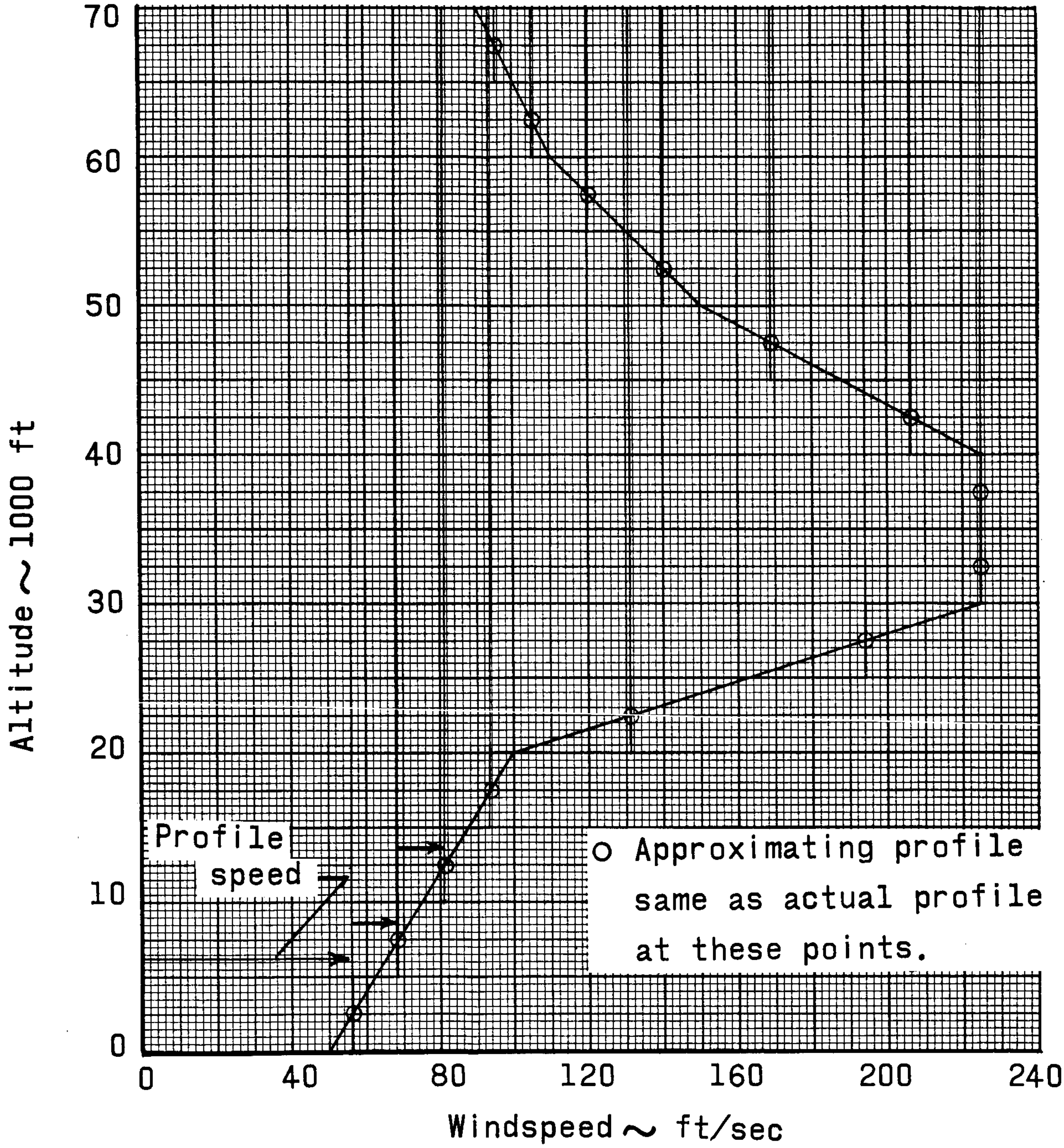

Figure 9.- Simulation of a wind profile.

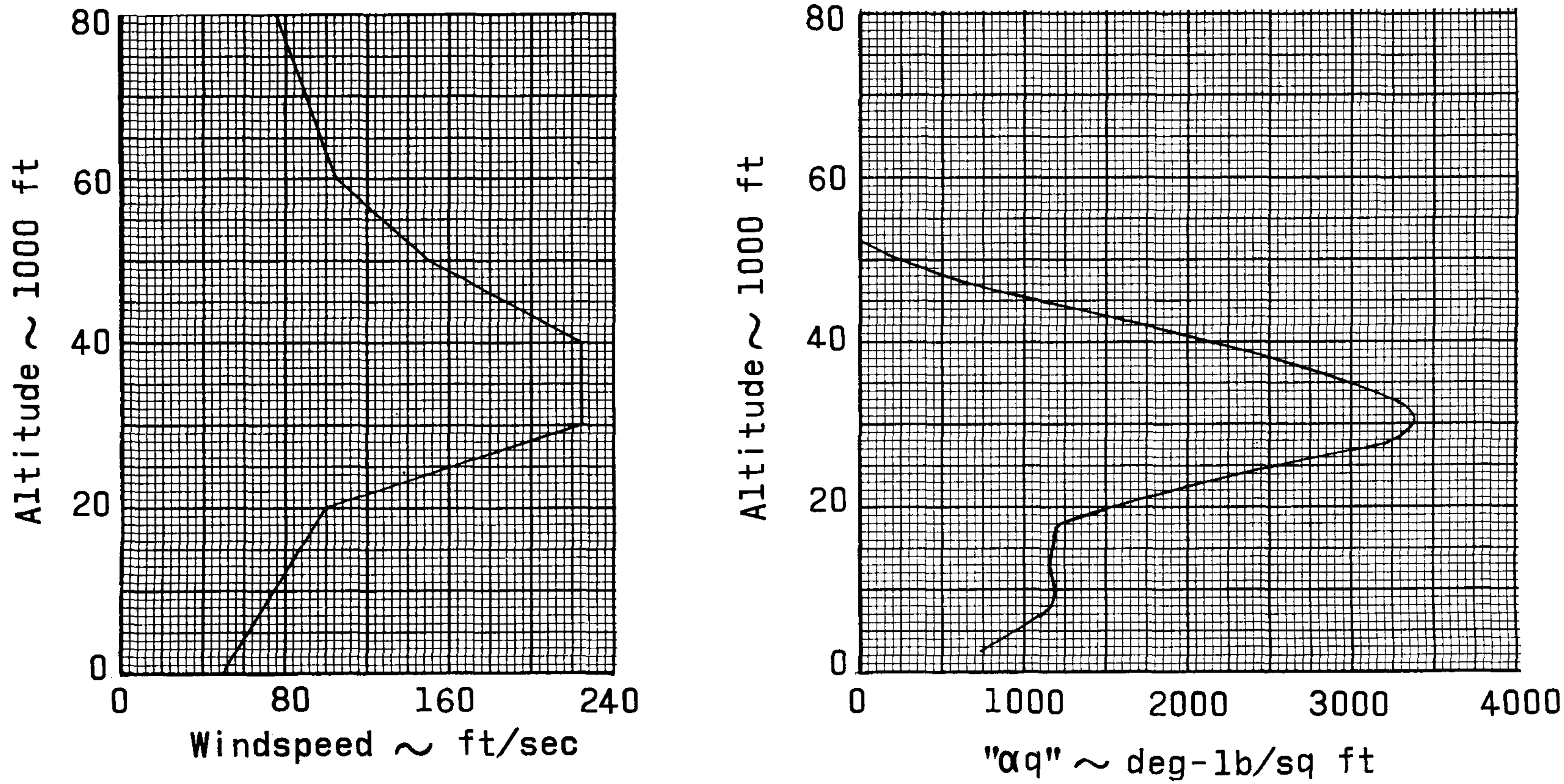

Figure 10.- C-1 response to a wind profile.